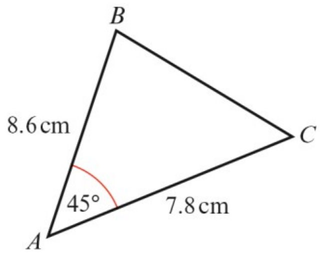
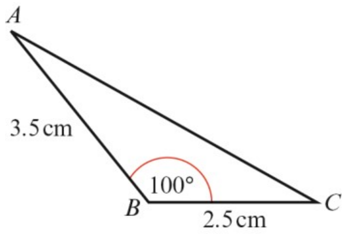
Exercise 6D

1 a



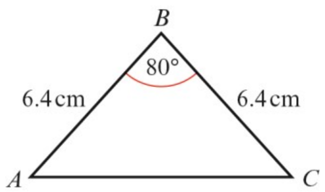
$\text{Area} = \frac{1}{2} \times 7.8 \times 8.6 \times \sin 45°$
$= 23.71\ldots$
$= 23.7 \text{ cm}^2$ (3 s.f.)

b



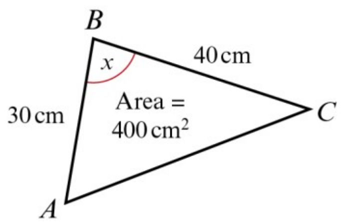
$\text{Area} = \frac{1}{2} \times 2.5 \times 3.5 \times \sin 100°$
$= 4.308\ldots$
$= 4.31 \text{ cm}^2$ (3 s.f.)

c



$\text{Area} = \frac{1}{2} \times 6.4 \times 6.4 \times \sin 80°$
$= 20.16\ldots$
$= 20.2 \text{ cm}^2$ (3 s.f.)

2 a



Using $\text{area} = \frac{1}{2} ac \sin B$

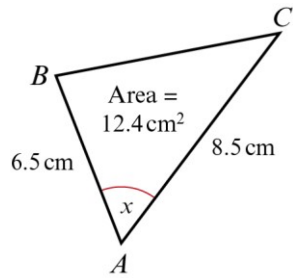
$400 = \frac{1}{2} \times 40 \times 30 \times \sin x$

2 a So $\sin x = \frac{400}{600} = \frac{2}{3}$

$x = \sin^{-1}\left(\frac{2}{3}\right)$ or $x = 180° - \sin^{-1}\left(\frac{2}{3}\right)$

$x = 41.8°$ (3 s.f.) or $x = 138°$ (3 s.f.)

b



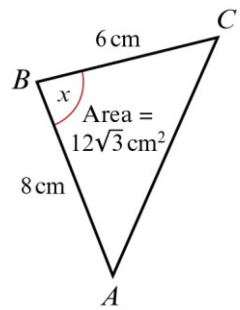
Using $\text{area} = \frac{1}{2} bc \sin A$

$12.4 = \frac{1}{2} \times 8.5 \times 6.5 \times \sin x$

So $\sin x = \frac{12.4}{27.625} = 0.04488\ldots$

$x = 26.7°$ (3 s.f) or $x = 153°$ (3 s.f.)

c



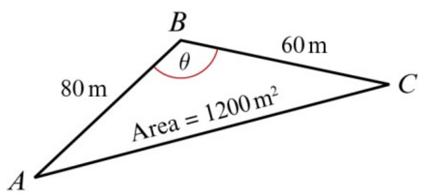
Using $\text{area} = \frac{1}{2} ac \sin B$

$12\sqrt{3} = \frac{1}{2} \times 6 \times 8 \sin x$

So $\sin x = \frac{12\sqrt{3}}{24} = \frac{\sqrt{3}}{2}$

$x = 60°$ or $x = 120°$

3



Using $\text{area} = \frac{1}{2} ac \sin B$

$1200 = \frac{1}{2} \times 60 \times 80 \times \sin \theta$

3 So $\sin\theta = \frac{1200}{2400} = \frac{1}{2}$

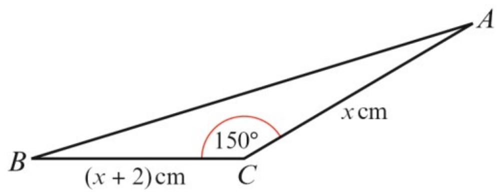
$$\theta = 30° \text{ or } \theta = 150°$$

But as AC is the largest side, θ must be the largest angle.
So $\theta = 150°$
Using the cosine rule to find AC:

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$AC^2 = 60^2 + 80^2 - 2\times 60\times 80\times\cos 150°$$
$$= 18\,313.84\ldots$$
$$AC = 135.3\ldots$$
$$= 135 \text{ m } (3 \text{ s.f.})$$

So the perimeter $= 60 + 80 + 135$
$= 275 \text{ m } (3 \text{ s.f.})$

4



Area of $\Delta ABC = \frac{1}{2}x(x+2)\sin 150°$

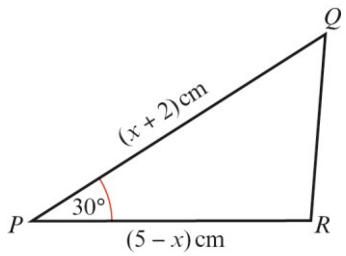
$$\text{So } 5 = \frac{1}{2}x(x+2)\times\frac{1}{2}$$
$$20 = x(x+2)$$

$\Rightarrow x^2 + 2x - 20 = 0$
Using the quadratic formula:

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-2\pm\sqrt{84}}{2}$$
$$x = 3.582\ldots \qquad \ldots$$

As $x > 0$, $x = 3.58$ cm (3 s.f.)

5



a Using area of $\Delta PQR = \frac{1}{2}qr\sin P$:

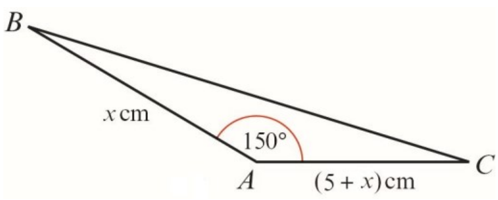
$$A = \frac{1}{2}(5-x)(x+2)\sin 30°$$
$$\Rightarrow A = \frac{1}{2}(5x - 2x + 10 - x^2)\times\frac{1}{2}$$
$$A = \frac{1}{4}(10 + 3x - x^2)$$

b Completing the square:

$$10 + 3x - x^2 = -\left(\left(x - 1\tfrac{1}{2}\right)^2 - 2\tfrac{1}{4} - 10\right)$$
$$= -\left(\left(x - 1\tfrac{1}{2}\right)^2 - 12\tfrac{1}{4}\right)$$
$$= 12\tfrac{1}{4} - \left(x - 1\tfrac{1}{2}\right)^2$$

When $x = 1\frac{1}{2}$:
The maximum value of $10 + 3x - x^2 = 12\frac{1}{4}$ and the maximum value of A is $\frac{1}{4}\left(12\frac{1}{4}\right) = 3\frac{1}{16}$
(You could use differentiation to find the maximum.)

6



a Using area of $\Delta BAC = \frac{1}{2}bc\sin A$

$$3\tfrac{3}{4} = \tfrac{1}{2}x(5+x)\sin 150°$$
$$3\tfrac{3}{4} = \tfrac{1}{2}(5x + x^2)\times\tfrac{1}{2}$$
$$15 = 5x + x^2$$
$$\Rightarrow x^2 + 5x - 15 = 0$$

b Using the quadratic formula:

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-5\pm\sqrt{85}}{2}$$
$$x = 2.109\ldots \qquad \ldots$$

As $x > 0$, $x = 2.11$ cm (3 s.f.)